

Relieving the Space Jam

Assessment of a Quick-Response Satellite Mission to Neutralize Debris
from Orbital Fragmentation Events

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Math Day — April 30th, 2016

Outline

- 1 The Mathematical Competition in Modeling
- 2 Space Junk
- 3 The Quick Response Mission Concept
- 4 Our Model
- 5 Results
- 6 Recommendations

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The Competition

What is the MCM?

- 3 person teams
- Choose 1 of 3 posed questions
- Answer the question to best of your abilities
- 4 days

Who does it?

- High school and college students
- 95 % foreign teams (mostly China)
- **Honorable Mention:** top 45% of 1453 teams

Problem B: Space Junk

"Develop a time-dependent model to determine the best alternative or combination of alternatives that a private firm could adopt as a commercial opportunity to address the space debris problem."

- Quantitative analysis of costs, risks, benefits, etc.
- Assess independent alternatives
- Address "What if?" scenarios
- Determine if an economically attractive opportunity exists
- Compare the different debris removal options
- Provide a specific recommendation as to how the debris should be removed
- Provide innovative alternatives for avoiding collisions
- Recommend a particular action, combination of actions, or no action

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What is Space Junk?

Thousands of man-made objects have reached space since the mid 1900's [3]

⇒ Uncontrolled debris when:

- End of functional lifespan
- Become unresponsive

⇒ Debris remains in orbit until gravity pulls it into Earth's atmosphere:

→ **Drag** → **Deceleration** → **De-orbited** → **DESTROYED!**

... but very slowly

⇒ Amount of debris is **rapidly growing!**

⇒ Many explosions and collisions, each one creating dust clouds with many smaller fragments traveling at very high velocities [5].

- Video: Individual Collision

<https://youtu.be/9cd0-4q0vb0?t=4m24s>

Why it's Bad

To illustrate, 1 kg object moving at normal low earth orbit speeds packs the punch of a 35,000 kg truck moving at 190 km/h [7]!

⇒ **Cascading effect**, as each piece of debris poses the risk of damage.

- Video: Domino Effect <https://youtu.be/9cd0-4q0vb0?t=5m44s>

⇒ At the end of the 20th century, over **35 million pieces of debris** were present in Earth's orbit [1].

- approximately 72% of cataloged debris is in the Lower Earth Orbit (< 2000 km [1])
- LEO contains the highest concentrations of debris at a density of 4.55×10^{-8} **debris fragments per cubic kilometer** and most concentrated around **900 km** altitude [10]

What to Do About It

- solar sail-type debris net
- gas puffs
- lasers
- and many other ideas...

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The Quick Response Mission Concept

The Idea

Launch a quick-response mission to target fragmentation and collision events soon after they occur.

The Idea:

- SORES: Space Orbit REmoval Satellite
- orbit in the same orbit that the satellite occupied, in the reverse direction
- deorbit all debris that it comes within a certain distance of (the "effective radius") through... some mechanism

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The Idea

Use standard classical physics concepts and Euler's method to propagate the position of an object over time to simulate its orbit.

- We approximate the system to a 1st order set of differential equations by updating a vector $U(t)$ of initial conditions.
- Apply the equation of force on mass m a distance d from a second mass M ,

$$F_g = GM \frac{m}{d^3} \vec{r}. \quad (1)$$

- Use Euler's method (explicit scheme) to solve the initial value problem of

$$\vec{x}' = F_g(t, U). \quad (2)$$

- Propagate the position and velocity over the time interval to trace the orbit of the object.

Modeling Foundation: Fragmentation Events

The Idea

We assume that all the energy generated in the fragmentation event is released as kinetic energy in random directions to equally sized fragments.

$$J = \sum_{i=0}^n KE_i = n \frac{1}{2} m_i \bar{v}_i^2. \quad (3)$$

Solving for \bar{v}_i yields

$$\bar{v}_i = \sqrt{2J/m_i}. \quad (4)$$

We assume that fragments disperse in random directions. Thus, with \vec{u}_i as a unit vector with random orientation, the velocity vector of the i th debris fragment, \vec{v}_i is given as

$$\vec{v}_i = \vec{v}_s + \bar{v}_i \vec{u}_i. \quad (5)$$

Amount of Debris

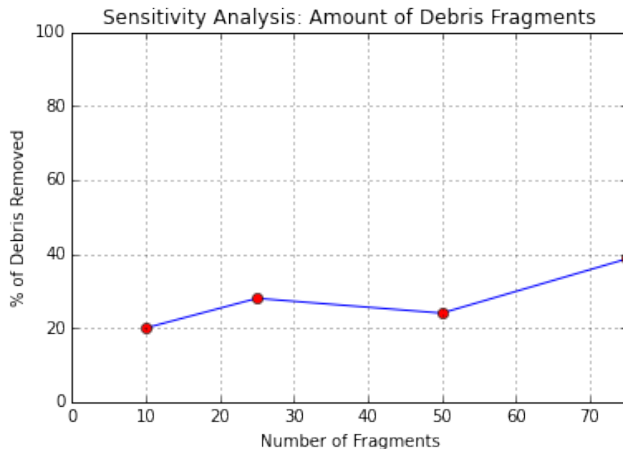


Figure 1: Results of sensitivity analysis of the performance of SORES predicted by our model to the number of debris generated in the orbital fragmentation event.

Explosion Energy

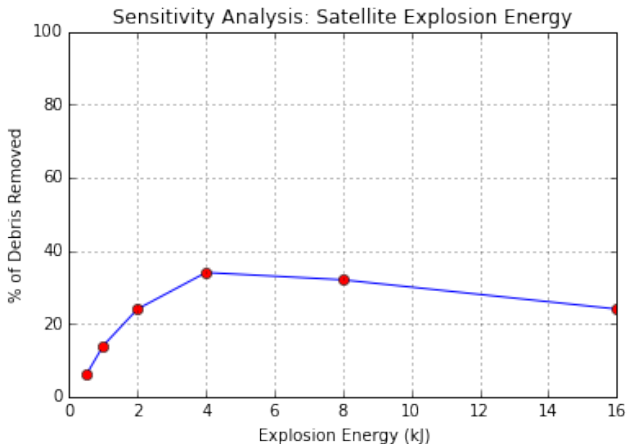


Figure 2: Results of sensitivity analysis of the performance of SORES predicted by our model to the energy released in the orbital fragmentation event.

Iterations of Numerical Approximation

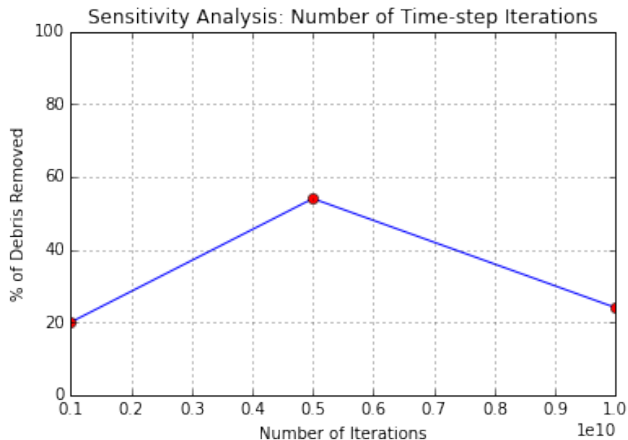


Figure 3: Results of sensitivity analysis of the performance of SORES predicted by our model to the number of iterations of each time-step.

Model Assessment: Visual

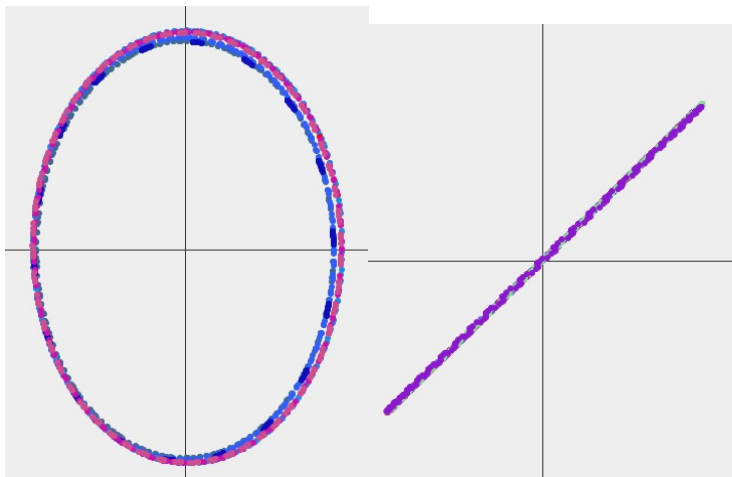


Figure 4: Side and top views of paths traced by orbital debris after a 2 kJ fragmentation event.

The Idea

Assess numerical approximation accuracy by looking at conservation of energy.

The total energy of each fragment can be given as

$$\begin{aligned} E_i &= KE_i + PE_i \\ &= \frac{1}{2} m_i \bar{v}_i^2 - \frac{GM_E m_i}{r} \end{aligned}$$

where m_i is the mass of the i th debris fragment.

- $N = 1 \times 10^8$ and $\Delta t = 7$ days $\rightarrow 2.0 \pm 1.7\%$ fluctuation in total energy of each fragment.
- $N = 1 \times 10^{10}$ and $\Delta t = 100$ $\rightarrow 11.9 \pm 2.7\%$ fluctuation in total energy of each fragment.

Model Assessment: Strengths

- The model simulates orbital mechanics to an adequate accuracy, as assessed by visual analysis of orbit path and mathematical evaluation of conservation of energy in the model.
- The model provides a simple, but reasonable, simulation of satellite fragmentation in three dimensions over a short time period.
- The model successfully assesses the number of debris de-orbited by a SORES satellite, with this success count responding to changes in parameters (such as SORES effective radius and delay before SORES launch) as would be expected.

Model Assessment: Weaknesses

- The model is computationally intensive, with computational costs scaling linearly both with number of fragments and mission duration. These costs make long-term simulation of large numbers of debris infeasible.
- The model neglects drag effects, the non-uniformity of Earth's gravitational field, and the gravitational interference of other celestial bodies. These effects are negligible for simulations of short time-periods, but their absence may affect longer term simulations.
- Most experimental results were derived from a single trial, which casts some uncertainty on their reproducibility.

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Time Delay to Launch

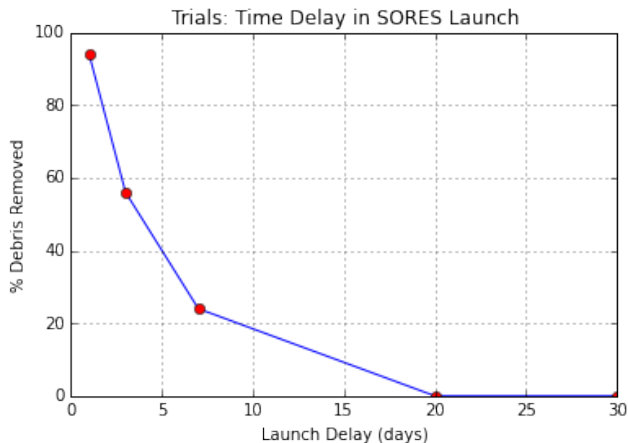


Figure 5: The performance of SORES, as measured by percent debris de-orbited, tabulated over several fragmentation-to-launch delays.

SORES Effective Radius

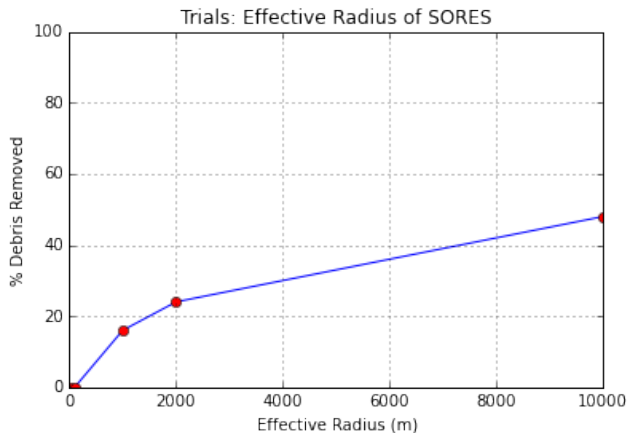


Figure 6: The relationship of SORES effective radius and its success at de-orbiting space debris.

Orbital Altitude

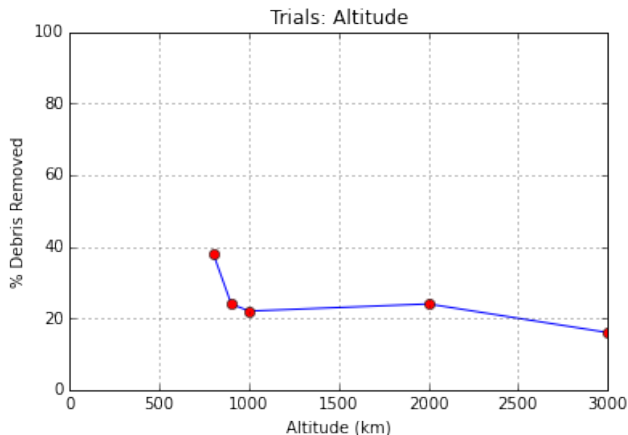


Figure 7: SORES performance tabulated over a orbital altitudes ranging from LEO to mid MEO.

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The Idea

A SORES approach to space junk removal simply does not pen out as a profitable endeavor at this time, but might be viable if costs go down and the space junk problem gets worse.

- Designing and launching a mission is expensive: \$290 million + [11]
- Insurance companies cover satellites for an average of \$129 million for catastrophic failure events, including damage from debris [11]

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Bounty of \$2,050 on each debris fragment, calculated using...

- The probability of a collision between a satellite and a debris fragment in a year
- The number of satellites with orbits passing through the LEO
- The approximate number of debris fragments in the LEO

⇒ 4880 debris fragments per mission

Recommendations

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- The effectiveness of a SORES mission also depends on the effective radius of debris capture. This radius should be, at a minimum, around 1 km.

Recommendations

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- To maximize the effectiveness of a SORES mission, minimizing the time that passes between a fragmentation event and launch is key.
- The effectiveness of a SORES mission also depends on the effective radius of debris capture. This radius should be, at a minimum, around 1 km.
- Our cost assessments suggest that a SORES-type intervention is not currently economically feasible. However, if the space debris situation deteriorates significantly, this type of intervention may become palatable.

For Further Reading I



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Acknowledgements

- Carl Toews
- UPS Math and Computer Science Department
- UPS Facilities

Model Assessment: Improvements

- Parallel computing techniques should be employed to reduce the computational costs of simulating our model.
- Experimental results should be derived from a battery of independent simulation trials.
- The model should be extended to specifically consider orbital collision events, in addition to orbital explosion events.
- The satellite fragmentation and orbital mechanics models should be compared to empirical data and be further developed and refined.