Modeling the Collective Behavior of Ants on Uneven Terrain

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Introduction

Motivation

Figure 1: Ant traffic
[Alexander Wild, a]

Figure 2: Human traffic
[Patrick T. Fallon, 2015]

Background

Figure 3: Video clip demonstrating route selection by foraging ants

Background

Figure 4: *Tetramorium caespitum* [Alexander Wild, c]

The collective foraging behavior of ants is well studied, including

- the strategies ants use to engage in foraging behavior [Camazine, 2003]
- how ants tend to select the shortest path to food [Camazine, 2003]
- how ants tend to select the richest food source [Camazine, 2003]
- approaches to mathematical modeling of ant foraging [Perna et al., 2012, Ryan, 2016]

Research Question

- **How does terrain affect the foraging path chosen by ants?**
- To travel between nest to food, do ants tend to select
  - the shortest path,
  - the quickest path,
  - some compromise between these, or
  - some other path all together?
- How might individual ant behaviors on uneven terrain contribute to collective decision making?

Figure 5: *Tetramorium caespitum* [Alexander Wild, b]

Approach

Experimental Design

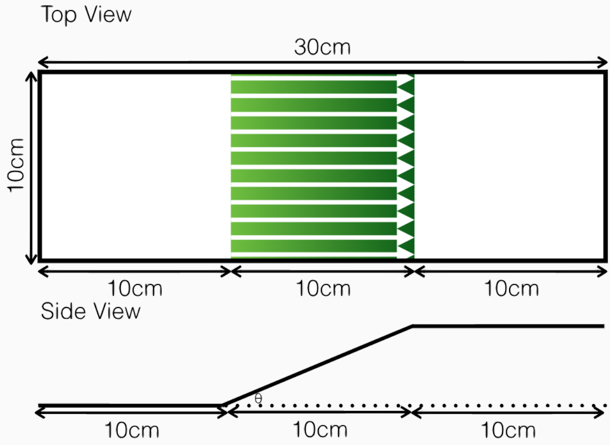

Figure 6: Arena terrain scheme

Experimental Design

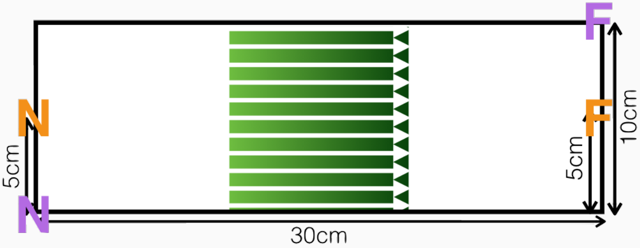

Figure 7: Nest and food placement scheme

Modeling Objectives

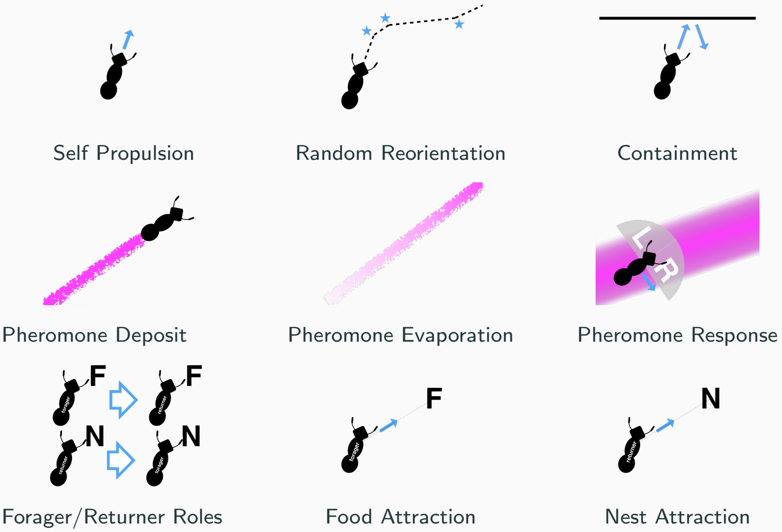

Figure 8: Major modeling considerations

Self Propulsion on Uneven Terrain

$$\frac{d}{dt}\begin{pmatrix}\vec{x}\\ \vec{v}\end{pmatrix} = \begin{pmatrix}\vec{v}\\ \hat{\vec{v}}\left[\frac{c}{\|\vec{v}\|} - a\|\vec{v}\| + \frac{\|\vec{v}\|^2 - b\vec{v}\cdot\nabla s}{\sqrt{\|\vec{v}\|^2 + (\vec{v}\cdot\nabla s)^2}}\right]\end{pmatrix}$$

- ants choose walking speed to expend constant power [Holt and Askew, 2012]
- gravity opposes uphill movement, aids downhill movement
- severe incline/decline decreases overall efficiency of ant movement

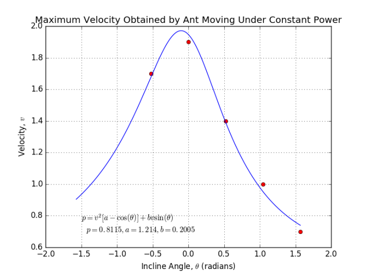

Figure 9: Ant velocity under constant power on inclined terrain

Random Reorientation Events [Khuong et al., 2013]

$$\frac{d}{dt}\begin{pmatrix}\vec{x}\\ \vec{v}\\ s\end{pmatrix} = \begin{pmatrix}\cdots\\ \cdots\\ \|v\|\end{pmatrix}$$

$$\theta_{\text{new}} = \theta_{\text{old}} + \boldsymbol{T}$$
$$s = 0$$
$$s_{\text{thresh}} = \boldsymbol{X}$$

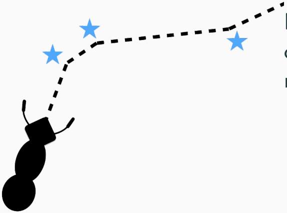

Figure 10: "Boltzmann walker" cartoon; blue stars denote random reorientation events

- upon reaching a threshold distance ($s > s_{\text{thresh}}$), the ant experiences a "reorientation event"
- the threshold distance is generated from an exponential distribution
- the angle the ant turns through is normally distributed

Random Reorientation Events: Adjustments

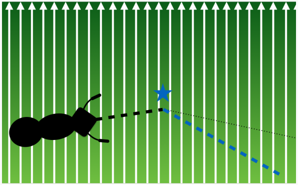

Figure 11: Illustration of adjustment accounting for ant behavior on uneven terrain

$$T_{\text{effective}} = T/\beta$$

$$\beta = \begin{cases} \text{forager role} & e^{c_1 p} \\ \text{returner role} & c_2 \end{cases}$$

$$s_{\text{thresh}} = X + c_3 \frac{|\vec{s} \cdot \vec{v}|}{\|\vec{v}\|}$$

- free path of ant (s_{thresh}) increases if ant oriented with or against the gradient [Khuong et al., 2013]
- ants preferentially re-orient themselves to align with or against a surface's topographical gradient [Khuong et al., 2013]
- severity of random reorientation decreased when following pheromone trail and returning to nest

Complete System

$$
\frac{d}{dt}\begin{pmatrix} \vec{x}_1 \\ \vec{v}_1 \\ s_1 \\ \vdots \\ p_1 \\ \vdots \end{pmatrix} = \begin{pmatrix} \vec{v}_1 \\ \alpha \hat{\vec{v}}_1 \left[\frac{c}{\|\vec{v}\|} - a\|\vec{v}\| + \frac{\|\vec{v}\|^2 - b\vec{v}\cdot\nabla s}{\sqrt{\|\vec{v}\|^2 + (\vec{v}\cdot\nabla s)^2}} \right] + \beta_{\vec{x}} \frac{\vec{a}-\vec{x}_1}{\|\vec{a}-\vec{x}_1\|} + \hat{\vec{v}}_{1\perp}(L_1 - R_1) + \gamma_{\vec{x}} \hat{\vec{v}}_{\perp} \left(\hat{\vec{v}}_{\perp} \cdot \frac{\vec{a}-\vec{x}}{\|\vec{a}-\vec{x}\|} \right) \\ \|\vec{v}_1\| \\ \vdots \\ \kappa f(p_1, \vec{x}_1, \ldots, \vec{x}_n) + \lambda p_1 \\ \vdots \end{pmatrix}
$$

events:

- out of bounds $\rightarrow$ reflect heading to "bounce" ant
- $s > s_{\text{thresh}} \rightarrow s = 0$, $s_{\text{thresh}} = \boldsymbol{X} + c_3 \frac{|\vec{s}\cdot\vec{v}|}{\|\vec{v}\|}$, random reorientation event with gradient alignment bias
- close to food/nest $\rightarrow$ switch forager/returner role

Animation

Figure 12: Animation of numerically-approximated solution

Results

Results (preliminary): Path Shape

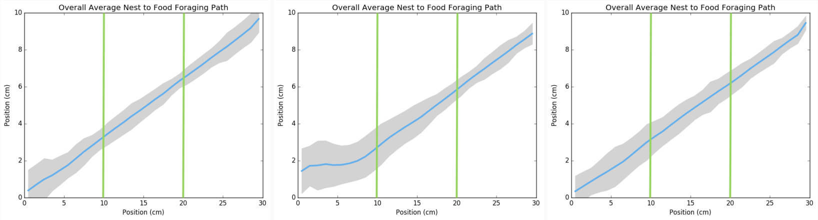

Figure 13: Comparison of overall average nest to food foraging path for, left to right, $-\pi/3$, 0, and $\pi/3$ radian inclines.

Results (preliminary): Path Length

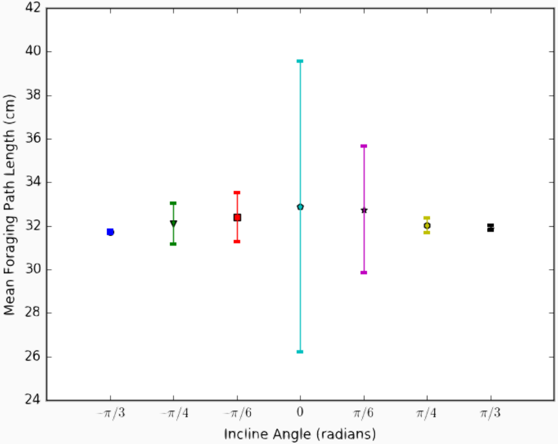

Figure 14: Comparison of path lengths over incline angles for corner-to-corner trials

Results (preliminary): Trip Duration

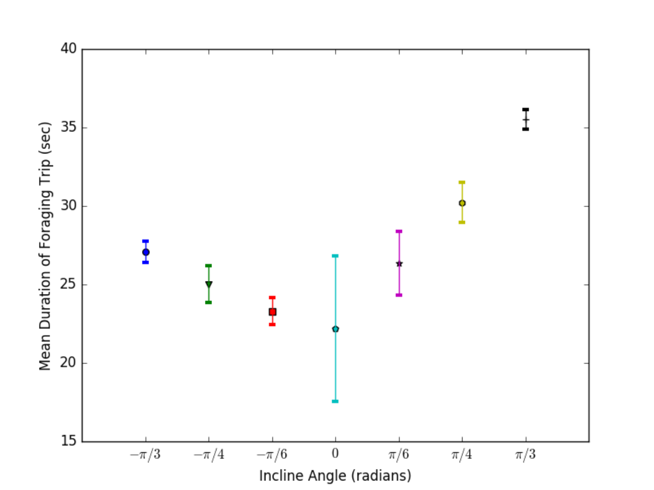

Figure 15: Comparison of trip durations over incline angles for corner-to-corner trials

Next Steps

- Refine model
    - variable pheromone deposition rate
- Perform further sensitivity analyses
    - pheromone grid granularity
    - pheromone sensitivity radius of ant
    - behavioral weighting
- **Compare model predictions with empirical results**

Figure 16: *Tetramorium caespitum* [Alexander Wild, d]

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mbi Mathematical Biosciences Institute

NJIT New Jersey Institute of Technology

The Ohio State University

Est. 1888 University of Puget Sound

Thank you!

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