Modeling the Collective Behavior of Ants on Uneven Terrain

Mathematical Biosciences Institute REU

Matthew Moreno[1], Jason Graham[2], Simon Garnier[3]

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[1]University of Puget Sound
[2]University of Scranton
[3]New Jersey Institute of Technology

Introduction

Motivation

Figure 1: Ant traffic
[Alexander Wild, a]

Figure 2: Human traffic
[Patrick T. Fallon, 2015]

Motivation

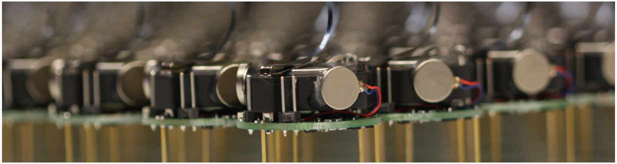

Figure 3: Kilobots, a common swarm robotics platform
[SSR Lab, Harvard,]

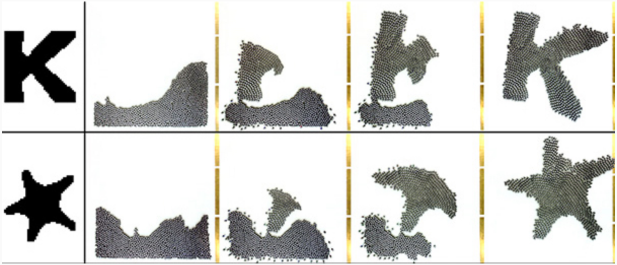

Figure 4: Kilobots in action
[Mike Rubenstein, 2014]

Background

Figure 5: Video clip of pheromone deposit and response by foraging ants

Background

Figure 6: Video clip demonstrating route selection by foraging ants

Background

Figure 7: *Tetramorium caespitum* [Alexander Wild, c]

The collective foraging behavior of ants is well studied, including

- the strategies ants use to engage in foraging behavior [Camazine, 2003]
- how ants tend to select the shortest path to food [Camazine, 2003]
- how ants tend to select the richest food source [Camazine, 2003]
- approaches to mathematical modeling of ant foraging [Perna et al., 2012, Ryan, 2016]

Research Question

- How does terrain affect the foraging path chosen by ants?

$$\Downarrow$$

- To travel between nest to food, do ants tend to select
    - the shortest path,
    - the quickest path,
    - some compromise between these, or
    - some other path all together?
- How might individual ant behaviors on uneven terrain contribute to collective decision making?

Approach

Experimental Design

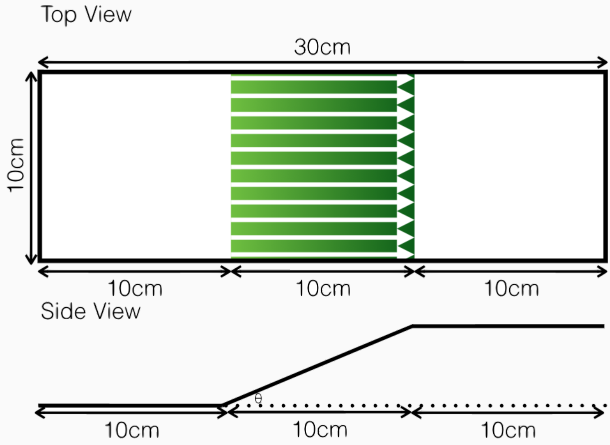

Figure 8: Arena terrain scheme

Experimental Design

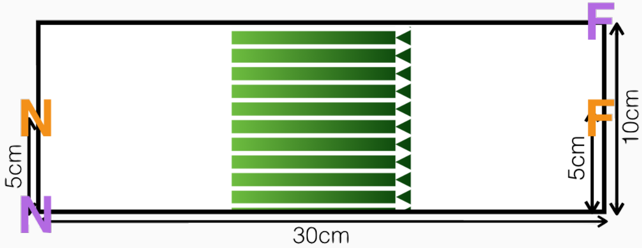

Figure 9: Nest and food placement scheme

Modeling Objectives

The model should consider:

- self-propulsion,
- containment in arena,
- forager/returner roles (attraction to food, attraction to nest),
- random reorientation events ("Boltzmann walker"),
- physical effects of gravity on inclined terrain,
- behavioral phenomena on inclined terrain,
- pheromone deposit, and
- pheromone response behavior.

System of Ordinary Differential Equations: Single Ant

$$\frac{d}{dt}\begin{pmatrix}\vec{x}\\ \vec{v}\end{pmatrix} = \begin{pmatrix}\vec{v}\\ \hat{\vec{v}}\left[\frac{c}{\|\vec{v}\|} - a\|\vec{v}\| + \frac{\|\vec{v}\|^2 - b\vec{v}\cdot\nabla s}{\sqrt{\|\vec{v}\|^2 + (\vec{v}\cdot\nabla s)^2}}\right]\end{pmatrix}$$

self-propulsion on uneven terrain:

- gravity opposes uphill movement, aids downhill movement
- sever incline/decline decreases overall efficiency of ant movement
- ants choose walking speed to expend constant power [Holt and Askew, 2012]

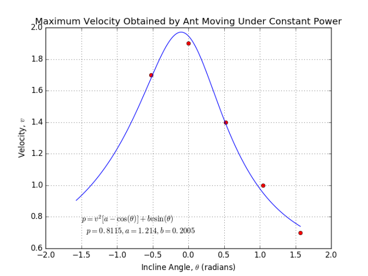

Figure 10: Ant velocity under constant power on inclined terrain

Events

- certain conditions trigger instantaneous changes in state variables
- example: bouncing ball

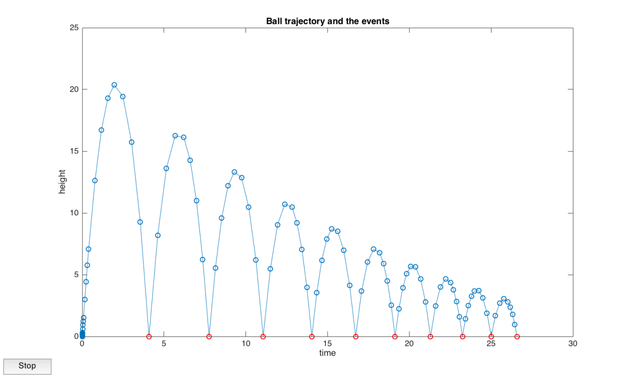

Figure 11: Matlab ballode example

Events

uses:

- "bouncing" ants off the arena walls
- switching between forager and returner roles
- random reorientation events

Events

$$\frac{d}{dt}\begin{pmatrix}\vec{x}\\ \vec{v}\\ s\end{pmatrix} = \begin{pmatrix}\dots\\ \dots\\ \|v\|\end{pmatrix}$$

$$\theta_{\text{new}} = \theta_{\text{old}} + \boldsymbol{T}$$
$$s = 0$$
$$s_{\text{thresh}} = \boldsymbol{X}$$

random reorientation events ("Boltzmann walker") [Khuong et al., 2013]:

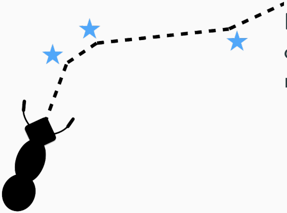

Figure 12: "Boltzmann walker" cartoon; blue stars denote random reorientation events.

- upon reaching a threshold distance ($s > s_{\text{thresh}}$), the ant experiences a "reorientation event"
- the threshold distance is generated from an exponential distribution
- the angle the ant turns through is normally distributed

Events

$$\boldsymbol{T}_{\text{effective}} = \boldsymbol{T}/\beta$$

$$\beta = \begin{cases} \text{forager role} & e^{c_1 p} \\ \text{returner role} & c_2 \end{cases}$$

$$s_{\text{thresh}} = \boldsymbol{X} + c_3 \frac{|\vec{s} \cdot \vec{v}|}{\|\vec{v}\|}$$

random reorientation events: adjustments

- free path of ant (s_{thresh}) increases if ant oriented with the gradient [Khuong et al., 2013]
- severity of random reorientation should decrease with
  - pheromone detection ("following trail")
  - returner status

Events

behavioral effect of incline:

- ants preferentially re-orient themselves to align with or against a surface's topographical gradient [Khuong et al., 2013]
- ants follow longer free paths when aligned with or against a surface's topographical gradient [Khuong et al., 2013]

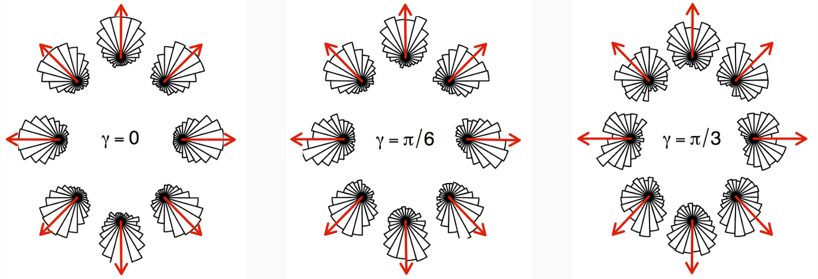

Figure 13: Ant Reorientation on Inclined Surfaces [Khuong et al., 2013]

System of Ordinary Differential Equations: Single Ant

effect of pheromone:

- ant accelerates perpendicular to its orientation
- magnitude of acceleration is proportional to the difference in concentration of pheromone over the “L” and “R” regions

$$\frac{d}{dt}\begin{pmatrix}\vec{x}\\ \vec{v}\end{pmatrix} = \begin{pmatrix}\cdots\\ \hat{\vec{v}}_{\perp}(L-R)\end{pmatrix}$$

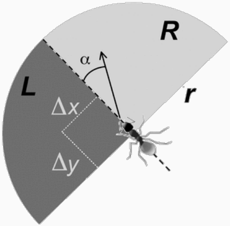

Figure 14: Regions of ant sensitivity to pheromone [Perna et al., 2012]

Complete System

putting it all together:

$$\frac{d}{dt}\begin{pmatrix}\vec{x}_1\\ \vec{v}_1\\ s_1\\ \vdots\\ p_1\\ \vdots\end{pmatrix} = \begin{pmatrix}\vec{v}_1\\ \alpha\hat{\vec{v}}_1\left[\frac{c}{\|\vec{v}\|} - a\|\vec{v}\| + \frac{\|\vec{v}\|^2 - b\vec{v}\cdot\nabla s}{\sqrt{\|\vec{v}\|^2+(\vec{v}\cdot\nabla s)^2}}\right] + \beta_{\vec{x}}\frac{\vec{a}-\vec{x}_1}{\|\vec{a}-\vec{x}_1\|} + \hat{\vec{v}}_{1\perp}(L_1 - R_1) + \gamma_{\vec{x}}\hat{\vec{v}}_{\perp}\left(\hat{\vec{v}}_{\perp}\cdot\frac{\vec{a}-\vec{x}}{\|\vec{a}-\vec{x}\|}\right)\\ \|\vec{v}_1\|\\ \vdots\\ \kappa f(p_1, \vec{x}_1, \ldots, \vec{x}_n) + \lambda p_1\\ \vdots\end{pmatrix}$$

events:

- out of bounds $\rightarrow$ reflect heading to "bounce" ant
- $s > s_{\text{thresh}} \rightarrow s = 0$, $s_{\text{thresh}} = \boldsymbol{X} + c_3\frac{|\vec{s}\cdot\vec{v}|}{\|\vec{v}\|}$, random reorientation event with gradient alignment bias
- close to food/nest $\rightarrow$ switch forager/returner role

Animation

Figure 15: Animation of numerically-approximated solution

Results

Results (preliminary): Path Shape

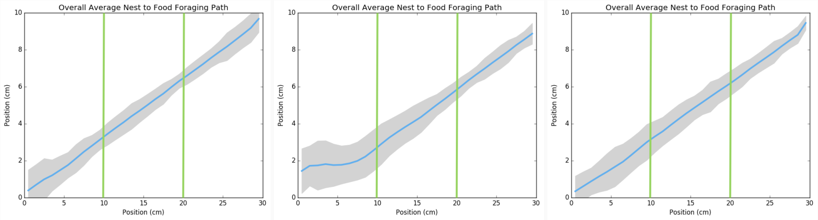

Figure 16: Comparison of overall average nest to food foraging path for, left to right, $-\pi/3$, 0, and $\pi/3$ radian inclines.

Results (preliminary): Path Length

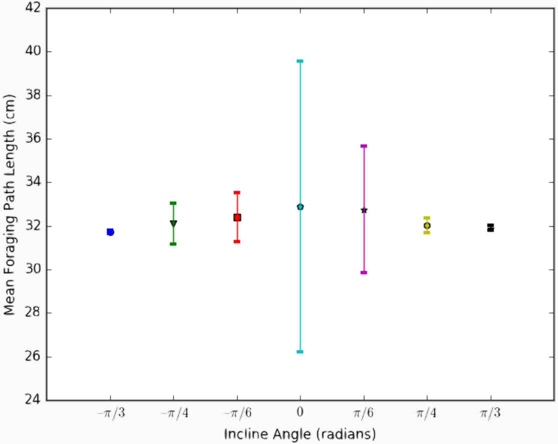

Figure 17: Comparison of path lengths over incline angles for corner-to-corner trials

Results (preliminary): Trip Duration

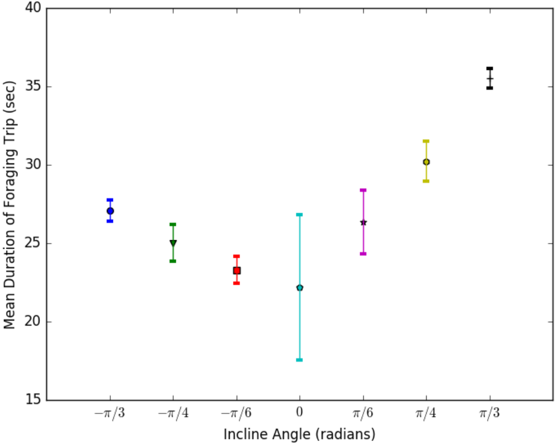

Figure 18: Comparison of trip durations over incline angles for corner-to-corner trials

Results (preliminary): Summary

- as expected, foraging trips take longer over steeper incline and decline
    - also taking longer over uphill versus downhill inclines

- the foraging path is generally more stable with steep incline or decline
    - ants are less likely to get lost/stuck
    - this effect is less pronounced in the center-to-center arena

- the foraging path becomes more direct with steeper incline or decline
    - even though the direct path is not aligned with the incline in the corner-to-corner arena
    - this effect is less pronounced in the center-to-center arena

Next Steps

- Refine model
  - variable pheromone deposition rate
- Perform further sensitivity analyses
  - pheromone grid granularity
  - pheromone sensitivity radius of ant
  - behavioral weighting
- Perform replicate simulations
- Compare model results with empirical results

Figure 19: *Tetramorium caespitum* [Alexander Wild, b]

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Questions?

References I

[Alexander Wild, a] Alexander Wild.
ant_battle1-XL.jpg.

[Alexander Wild, b] Alexander Wild.
ant_battle2-XL.jpg.

[Alexander Wild, c] Alexander Wild.
caespitum-16j-XL.jpg.

[Camazine, 2003] Camazine, S. (2003).
Self-organization in biological systems.
Princeton studies in complexity. Princeton University Press, Princeton, N.J Woodstock.

References II

[Holt and Askew, 2012] Holt, N. C. and Askew, G. N. (2012).
Locomotion on a slope in leaf-cutter ants: metabolic energy use, behavioural adaptations and the implications for route selection on hilly terrain.
Journal of Experimental Biology, 215(15):2545–2550.

[Khuong et al., 2013] Khuong, A., Lecheval, V., Fournier, R., Blanco, S., Weitz, S., Bezian, J.-J., and Gautrais, J. (2013).
How Do Ants Make Sense of Gravity? A Boltzmann Walker Analysis of Lasius niger Trajectories on Various Inclines.
PLoS ONE, 8(10):e76531.

[Mike Rubenstein, 2014] Mike Rubenstein (2014).
Kilobots31.

[Patrick T. Fallon, 2015] Patrick T. Fallon (2015).
960x0.jpg.

References III

[Perna et al., 2012] Perna, A., Granovskiy, B., Garnier, S., Nicolis, S. C., Labdan, M., Theraulaz, G., Fourcassi, V., and Sumpter, D. J. T. (2012).
Individual Rules for Trail Pattern Formation in Argentine Ants (Linepithema humile).
PLoS Computational Biology, 8(7):e1002592.

[Ryan, 2016] Ryan, S. D. (2016).
A model for collective dynamics in ant raids.
Journal of Mathematical Biology, 72(6):1579–1606.

[SSR Lab, Harvard,] SSR Lab, Harvard.
swarm2.jpg.